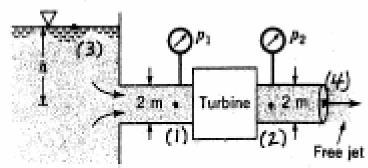
5.119

5.119 The turbine shown in Fig. P5.119 develops 2500 kW when the water flowrate is 20 m³/s. The head loss across the turbine from (1) to (2) is negligible, but the head loss for the entire flow is 2.5 m. (a) Determine the pressure difference, $p_1 - p_2$, across the turbine. (b) Determine the elevation h.



■ FIGURE P5.119

(a) The energy equation across the turbine is

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_{L_{1-2}} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \text{ where } z_1 = z_2,\ h_{L_{1-2}} = 0,\ V_1 = V_2$$

Thus,

$$\frac{p_1}{\gamma} + h_s = \frac{p_2}{\gamma} \text{ or}$$

$$p_1 - p_2 = -\gamma h_s, \text{ where } h_s = \frac{\dot{W}_s}{\gamma Q}$$

Hence,

$$p_1 - p_2 = -\gamma\left(\frac{\dot{W}_s}{\gamma Q}\right) = -\frac{\dot{W}_s}{Q} = -\left(\frac{-2500 \times 10^3 \frac{N\cdot m}{s}}{20 \frac{m^3}{s}}\right) = 125 \times 10^3 \frac{N}{m^2} = \underline{\underline{125 \text{ kPa}}}$$

(Note: $\dot{W}_s < 0$ because a turbine removes energy from the fluid.)

(b) Also, from (3) to (4)

$$\frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g} + h_s - h_{L_{3-4}} = \frac{p_4}{\gamma} + z_4 + \frac{V_4^2}{2g}, \text{ where } p_3 = p_4 = 0,\ z_4 = 0,\ V_3 = 0, \text{ and } z_3 = h$$

Thus,

$$h + h_s - h_{L_{3-4}} = \frac{V_4^2}{2g}, \text{ or}$$

$$h = \frac{V_4^2}{2g} + h_{L_{3-4}} - h_s \qquad (1)$$

$$\text{Also, } V_4 = \frac{Q}{A_4} = \frac{20 \frac{m^3}{s}}{\left(\frac{\pi}{4}(2m)^2\right)} = 6.37 \frac{m}{s},\ h_{L_{3-4}} = 2.5m, \text{ and}$$

$$h_s = \frac{\dot{W}_s}{\gamma Q} = \frac{-2500 \times 10^3 \frac{N\cdot m}{s}}{\left(9.83 \times 10^3 \frac{N}{m^3}\right)\left(20 \frac{m^3}{s}\right)} = -12.76 m$$

Therefore, from Eq. (1)

$$h = \frac{\left(6.37 \frac{m}{s}\right)^2}{2\left(9.81 \frac{m}{s^2}\right)} + 2.5m - (-12.76m) = \underline{\underline{17.3 \text{ m}}}$$

5.124

5.124 The velocity profile in a turbulent pipe flow may be approximated with the expression

$$\frac{u}{u_c} = \left(\frac{R-r}{R}\right)^{1/n}$$

where u = local velocity in the axial direction, u_c = centerline velocity in the axial direction, R = pipe inner radius from pipe axis, r = local radius from pipe axis, and n = constant. Determine the kinetic energy coefficient, α, for: **(a)** $n = 5$; **(b)** $n = 6$; **(c)** $n = 7$; **(d)** $n = 8$; **(e)** $n = 9$; **(f)** $n = 10$.

For the kinetic energy coefficient, α, we may use Eq. 5.86. Thus,

$$\alpha = \frac{\int_0^R \frac{u^2}{2}\rho u\, 2\pi r\, dr}{\rho \bar{u}\, \pi R^2 \frac{\bar{u}^2}{2}} = \frac{2\int_0^1 u^3\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)}{\bar{u}^3} = \frac{2u_c^3\int_0^1 \left(1-\frac{r}{R}\right)^{\frac{3}{n}}\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)}{\bar{u}^3} \qquad (1)$$

For the average velocity, $\bar{u}$, we may use Eq. 5.7. Thus,

$$\bar{u} = \frac{\int_0^R \rho u\, 2\pi r\, dr}{\rho \pi R^2} = 2\int_0^1 u\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = 2u_c\int_0^1 \left(1-\frac{r}{R}\right)^{\frac{1}{n}}\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) \qquad (2)$$

To facilitate the integrations we make the substitution

$$\beta = 1 - \frac{r}{R} \qquad (3)$$

Thus,

$$d\beta = -d\left(\frac{r}{R}\right) \qquad (4)$$

and Eq. 2 becomes

$$\bar{u} = -2u_c\int_1^0 \beta^{\frac{1}{n}}(1-\beta)\, d\beta = \frac{2n^2}{(n+1)(2n+1)}u_c \qquad (5)$$

Combining Eqs. 1, 3, 4 and 5 we obtain

$$\alpha = \frac{-2\int_1^0 \beta^{\frac{3}{n}}(1-\beta)\, d\beta}{\left[\frac{2n^2}{(n+1)(2n+1)}\right]^3} = \left[\frac{2n^2}{(3+n)(3+2n)}\right]\left[\frac{(n+1)(2n+1)}{2n^2}\right]^3 \qquad (6)$$

(a) For $n = 5$, Eq. 6 yields

$$\alpha = \left\{\frac{2(5)^2}{(3+5)[3+2(5)]}\right\}\left\{\frac{(5+1)[(2)(5)+1]}{2(5)^2}\right\}^3 = \underline{\underline{1.11}}$$

(d) For $n = 8$

$\underline{\underline{\alpha = 1.05}}$

(e) For $n = 9$

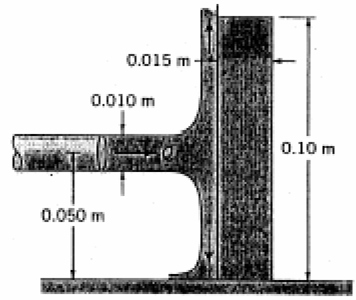
$\alpha = \underline{\underline{1.04}}$

(f) For $n = 10$

$\alpha = \underline{\underline{1.03}}$

5.29

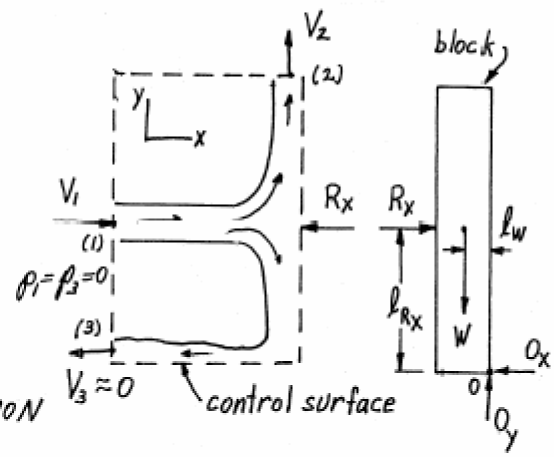
5.29 A 10-mm diameter jet of water is deflected by a homogeneous rectangular block (15 mm by 200 mm by 100 mm) that weighs 6 N as shown in Video V5.4 and Fig. P5.29 Determine the minimum volume flowrate needed to tip the block.



■ FIGURE P5.29

From the free body diagram of the block when it is ready to tip $\sum M_0 = 0$, or

$R_x \ell_{R_x} = W \ell_w$ where R_x is the fore that the water puts on the block.



Thus,

$$R_x = \frac{W \ell_w}{\ell_{R_x}} = \frac{6\,N \left(\frac{0.015}{2} m\right)}{0.050 m} = 0.90 N$$

For the control volume shown the x-component of the momentum equation

$$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = \sum F_x$$

becomes

$$V_1 \rho (-V_1) A_1 = -R_x \quad \text{or} \quad V_1 = \sqrt{\frac{R_x}{\rho A_1}}$$

Thus,

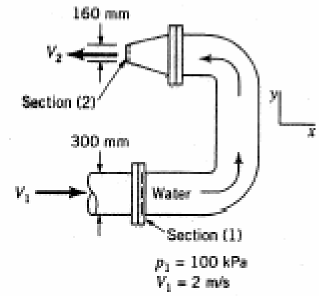
$$V_1 = \sqrt{\frac{0.9 N}{\left(999 \frac{kg}{m^3}\right) \frac{\pi}{4} (0.01 m)^2}} = 3.39 \frac{m}{s}$$

Hence,

$$Q = A_1 V_1 = \frac{\pi}{4} (0.01 m)^2 \left(3.39 \frac{m}{s}\right) = \underline{\underline{2.66 \times 10^{-4} \frac{m^3}{s}}}$$

5.32

5.32 Determine the magnitude and direction of the anchoring force needed to hold the horizontal elbow and nozzle combination shown in Fig. P5.32 in place. Atmospheric pressure is 100



■ FIGURE P5.32

The control volume shown in the sketch above is used. Application of the y direction component of the linear momentum equation yields

$$R_y = \underline{\underline{0}}$$

Application of the x direction linear momentum equation leads to

$$-u_1 \rho u_1 A_1 - u_2 \rho u_2 A_2 = p_1 A_1 - R_x + p_2 A_2$$

From the conservation of mass equation

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2$$

Thus

$$R_x = \rho u_1 A_1 \left(u_1 + u_2\right) + p_1 A_1 + p_2 A_2 = \rho u_1 \frac{\pi D_1^2}{4}\left(u_1 + \frac{D_1^2}{D_2^2} u_1\right) + p_1 \frac{\pi D_1^2}{4} + (0) A_2$$

or

$$R_x = \left(999 \frac{kg}{m^3}\right)\left(2 \frac{m}{s}\right)\frac{\pi}{4}\frac{(300\ mm)^2}{\left(1000 \frac{mm}{m}\right)^2}\left[\left(2 \frac{m}{s}\right) + \frac{(300\ mm)^2}{(160}\left(2 \frac{m}{s}\right)\right]$$

$$+ (100\ kPa)\frac{\pi}{4}\frac{(300\ mm)^2}{\left(1000 \frac{mm}{m}\right)^2}\left(1000 \frac{N}{m^2 \cdot kPa}\right)$$

and

$$R_x = \underline{\underline{8340\ N}}$$

5.34

5.34 A converging elbow (see Fig. P5.34) turns water through an angle of 135° in a vertical plane. The flow cross section diameter is 400 mm at the elbow inlet, section (1), and 200 mm at the elbow outlet, section (2). The elbow flow passage volume is 0.2 m³ between sections (1) and (2). The water volume flowrate is 0.4 m³/s and the elbow inlet and outlet pressures are 150 kPa and 90 kPa. The elbow mass is 12 kg. Calculate the horizontal (x direction) and vertical (z direction) anchoring forces required to hold the elbow in place.

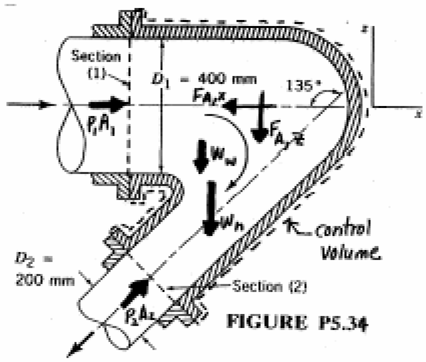


FIGURE P5.34

A control volume that contains the elbow and the water within the elbow between sections (1) and (2) as shown in the sketch above is used. Application of the horizontal or x direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 - V_2 \cos 45^\circ \rho V_2 A_2 = p_1 A_1 - F_{A,x} + p_2 A_2 \cos 45^\circ$$

From conservation of mass

$$\dot{m} = \rho u_1 A_1 = \rho V_2 A_2 = \rho Q \qquad (1)$$

Thus

$$F_{A,x} = \frac{\rho Q^2}{A_1} + \frac{\rho Q^2}{A_2}\cos 45^\circ + p_1 A_1 + p_2 A_2 \cos 45^\circ = \frac{\rho Q^2}{\frac{\pi D_1^2}{4}} + \frac{\rho Q^2 \cos 45^\circ}{\frac{\pi D_2^2}{4}} + p_1 \frac{\pi}{4} D_1^2 + p_2 \frac{\pi}{4} D_2^2 \cos 45^\circ$$

$$F_{A,x} = \left(999 \frac{kg}{m^3}\right)\left(0.4 \frac{m^3}{s}\right)^2 \frac{1}{\left(\frac{\pi}{4}\right)} \left[\frac{\left(1000 \frac{mm}{m}\right)^2}{(400\ mm)^2} + \frac{\cos 45^\circ \left(1000 \frac{mm}{m}\right)^2}{(200\ mm)^2}\right]\left(\frac{1\ N}{kg \frac{m}{s^2}}\right)$$
$$+ \frac{\pi}{4} \frac{\left(1000 \frac{N}{kPa\ m^2}\right)}{\left(1000 \frac{mm}{m}\right)^2}\left[(150\ kPa)(400\ mm)^2 + (90\ kPa)(200\ mm)^2 \cos 45^\circ\right]$$

$$F_{A,x} = \underline{\underline{25,700\ N}}$$

Application of the vertical or z direction component of the linear momentum equation leads to

$$-V_2 \sin 45^\circ \rho V_2 A_2 = p_2 A_2 \sin 45^\circ - F_{A,z} - W_w - W_e$$

which when combined with Eq. 1 gives

$$F_{A,z} = \frac{\rho Q^2}{A_2}\sin 45^\circ + p_2 A_2 \sin 45^\circ - W_w - W_e = \frac{\rho Q^2}{\frac{\pi D_2^2}{4}}\sin 45^\circ + p_2 \frac{\pi D_2^2}{4}\sin 45^\circ - \rho g \forall_w - m_e g$$

(con't)

5.34	(con't)

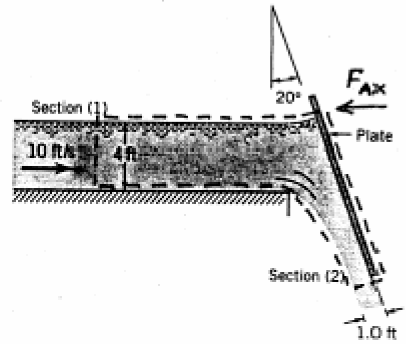
$$F_{A,z} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(0.4\,\frac{m^3}{s}\right)^2 \sin 45^\circ}{\frac{\pi}{4}\frac{(200\,mm)^2}{\left(1000\,\frac{mm}{m}\right)^2}}\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) + \frac{(90\,kPa)\,\pi\,(200\,mm)^2 \sin 45^\circ}{4\left(1000\,\frac{mm}{m}\right)^2}$$

$$-\left(999\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)\left(0.2\,m^3\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) - (12\,kg)\left(9.81\,\frac{m}{s^2}\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)$$

$$F_{A,z} = \underline{\underline{8920\,N}}$$

5.48

5.48 Water flows from a two-dimensional open channel and is diverted by an inclined plate as illustrated in Fig. P5.48. When the velocity at section (1) is 10 ft/s, what horizontal force (per unit width) is required to hold the plate in position? At section (1) the pressure distribution is hydrostatic, and the fluid acts as a free jet at section (2). Neglect friction.



■ FIGURE P5.48

A control volume that contains most of the plate and the water being turned by the plate as shown in the sketch above is used. Application of the horizontal x-direction component of the linear momentum equation yields

$$-V_1 \rho V_1 A_1 + V_2 \sin 20° \rho V_2 A_2 = -F_{Ax} + \frac{1}{2}\gamma_w h_1 A_1 \qquad (1)$$

From conservation of mass we obtain

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{h_1}{h_2} V_1$$

Thus, Eq. 1 becomes for unit width

$$-V_1^2 \rho h_1 + \left(\frac{h_1}{h_2} V_1\right)^2 \sin 20° \rho h_2 = -F_{Ax} + \frac{1}{2}\gamma_w h_1^2$$

or

$$F_{Ax} = \frac{1}{2}\gamma_w h_1^2 + V_1^2 \rho h_1 - \left(\frac{h_1}{h_2} V_1\right)^2 \sin 20° \rho h_2$$

Then

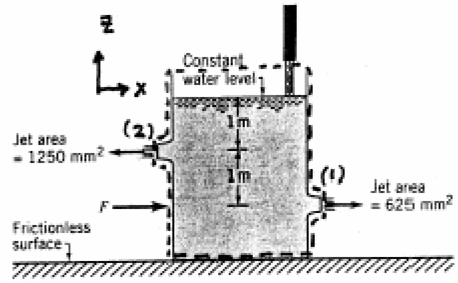
$$F_{Ax} = \frac{1}{2}\left(62.4 \frac{lb}{ft^3}\right)(4\ ft)^2 + \left(10\ \frac{ft}{s}\right)^2\left(1.94 \frac{slugs}{ft^3}\right)\left(1\ \frac{lb \cdot s^2}{slug \cdot ft}\right)(4\ ft)$$
$$- \left[\left(\frac{4\ ft}{1\ ft}\right)\left(10\ \frac{ft}{s}\right)\right]^2 \sin 20° \left(1.94 \frac{slugs}{ft^3}\right)\left(1\ \frac{lb \cdot s^2}{slug \cdot ft}\right)(1\ ft)$$

and

$$F_{Ax} = \underline{\underline{213}}\ lb$$

5.56

5.56 Water is added to the tank shown in Fig. P5.56 through a vertical pipe to maintain a constant (water) level. The tank is placed on a horizontal plane which has a frictionless surface. Determine the horizontal force, F, required to hold the tank stationary. Neglect all losses.



■ FIGURE P5.56

Applying the x-direction component of the linear momentum equation to the contents of the control volume sketched above we get

$$V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 = F \qquad (1)$$

Using Bernoulli's equation to describe the frictionless flow from the constant water surface level to the flow leaving at stations (1) and (2) we obtain

$$V_2 = \sqrt{2 g h_2} \qquad (2)$$

and

$$V_1 = \sqrt{2 g h_1} \qquad (3)$$

Combining Eqs. 1, 2 and 3 we get

$$F = 2 g h_1 \rho A_1 - 2 g h_2 \rho A_2$$

or

$$F = 2\left(9.81 \frac{m}{s^2}\right)\left(999 \frac{kg}{m^3}\right)\left[\frac{(2\, m)(625\, mm^2)}{\left(1000 \frac{mm}{m}\right)^2} - \frac{(1\, m)(1250\, mm^2)}{\left(1000 \frac{mm}{m}\right)^2}\right]$$

and

$$F = \underline{\underline{0\ N}}$$

5.121

5.121 Water is to be moved from one large reservoir to another at a higher elevation as indicated in Fig. P5.121. The loss in available energy associated with 2.5 ft³/s being pumped from sections (1) to (2) is $61\overline{V}^2/2$ where $\overline{V}$ is the average velocity of water in the 8-in.-inside diameter piping involved. Determine the amount of shaft power required.

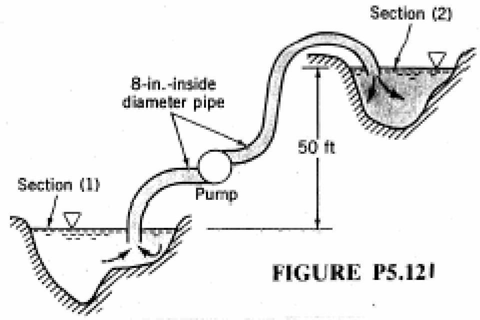


FIGURE P5.121

For the flow from section (1) to section (2) Eq. 5.82 leads to

$$\dot{W}_{\substack{shaft\\net\ in}} = \rho Q\left[g(z_2 - z_1) + loss\right] = \rho Q\left[g(z_2-z_1) + 61\frac{\overline{V}^2}{2}\right] \quad (1)$$

From the volume flowrate we obtain

$$\overline{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = \frac{\left(2.5\ \frac{ft^3}{s}\right)}{\frac{\pi}{4}\left(\frac{8\ in.}{12\ \frac{in.}{ft}}\right)^2} = 7.162\ \frac{ft}{s}$$

Thus, from Eq. 1

$$\dot{W}_{\substack{shaft\\net\ in}} = \left(1.94\ \frac{slugs}{ft^3}\right)\left(2.5\ \frac{ft^3}{s}\right)\left[\left(32.2\ \frac{ft}{s^2}\right)(50\ ft) + \frac{(61)\left(7.162\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\left(\frac{1}{550\ \frac{ft\cdot lb}{s\cdot hp}}\right)$$

or

$$\dot{W}_{\substack{shaft\\net\ in}} = \underline{\underline{28\ hp}}$$

5.123

5.123 Water is to be pumped from the large tank shown in Fig. P5.123 with an exit velocity of 6 m/s. It was determined that the original pump (pump 1) that supplies 1 kW of power to the water did not produce the desired velocity. Hence, it is proposed that an additional pump (pump 2) be installed as indicated to increase the flowrate to the desired value. How much power must pump 2 add to the water? The head loss for this flow is $h_L = 250Q^2$, where h_L is in m when Q is in m^3/s.

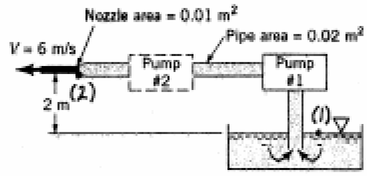


FIGURE P5.123

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = p_2 = 0,\ V_1 = 0,\ z_1 = 0,\ z_2 = 2m.$

Thus,

$h_p = h_L + z_2 + \frac{V_2^2}{2g}$, where $V_2 = 6 m/s$ so that $Q = A_2 V_2 = 0.01 m^2 (6 m/s)$
$= 0.06 \frac{m^3}{s}$

Note: $h_p = h_{pump1} + h_{pump2}$

Thus, with $h_L = 250Q^2 = 250(0.06)^2 = 0.90m$ it follows that

$$h_p = 0.90m + 2m + \frac{(6 m/s)^2}{2(9.81 \frac{m}{s^2})} = 4.73m$$

so that

$$\dot{W}_p = \gamma Q h_p = (9.80 \times 10^3 \frac{N}{m^3})(0.06 \frac{m^3}{s})(4.73m) = 2.78 \times 10^3 \frac{N \cdot m}{s} = 2.78\ kW$$

Therefore,

$\dot{W}_p = \dot{W}_{pump1} + \dot{W}_{pump2} = 2.78\ kW$, with $\dot{W}_{pump1} = 1\ kW$

Hence,

$$\dot{W}_{pump2} = 2.78\ kW - 1\ kW = \underline{\underline{1.78\ kW}}$$

5.28

5.28 Water flows through a horizontal, 180° pipe bend as is illustrated in Fig. P5.28. The flow cross section area is constant at a value of 9000 mm^2. The flow velocity everywhere in the bend is 15 m/s. The pressures at the entrance and exit of the bend are 210 and 165 kPa, respectively. Calculate the horizontal (x and y) components of the anchoring force needed to hold the bend in place.

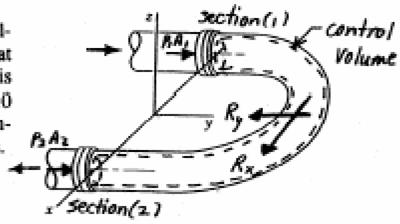


FIGURE P5.28

This analysis is similar to the one of Example 5.11. A fixed, non-deforming control volume that contains the water within the elbow between sections (1) and (2) at an instant is used. The horizontal forces acting on the contents of the control volume in the x and y directions are shown. Application of the x-direction component of the linear momentum equation (Eq. 5.22) leads to

$$R_x = \underline{\underline{0}}$$

Application of the y-direction component of the linear momentum equation yields

$$-v_1 \rho v_1 A_1 - v_2 \rho v_2 A_2 = p_1 A_1 - R_y + p_2 A_2$$

or

$$R_y = \rho A_1 v_1 (v_1 + v_2) + p_1 A_1 + p_2 A_2$$

Thus

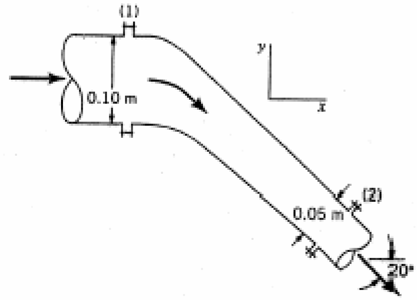
$$R_y = \left(999 \frac{kg}{m^3}\right)\frac{(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2}\left(15 \frac{m}{s}\right)\left(15\frac{m}{s} + 15\frac{m}{s}\right)\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right) + \frac{(210\ kPa)(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1}{1000 \frac{N}{m^2 \cdot kPa}}\right)}$$

$$+ \frac{(165\ kPa)(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2 \left(\frac{1}{1000 \frac{N}{m^2 \cdot kPa}}\right)}$$

$$R_y = \underline{\underline{7420}}\ N$$

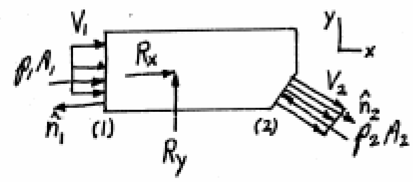
5.31

5.31 Water flows through the 20° reducing bend shown in Fig. P5.31 at a rate of 0.025 m^3/s. The flow is frictionless, gravitational effects are negligible, and the pressure at section (1) is 150 kPa. Determine the x and y components of force required to hold the bend in place.



■ FIGURE P5.31

For the control volume shown the x-component of the momentum equation is



$$\int_{cs} u\rho\vec{V}\cdot\hat{n}\,dA = \Sigma F_x \text{, or}$$

$$V_1\rho(-V_1)A_1 + (V_2\cos 20°)\rho V_2 A_2 = R_x + p_1A_1 - p_2A_2\cos 20°$$

or

(1) $$R_x = p_2A_2\cos 20° - p_1A_1 + (V_2\cos 20° - V_1)\dot{m}, \text{ where } \dot{m} = \dot{m}_1 = \dot{m}_2 = \rho Q = \rho AV$$

Also,

$$V_1 = Q/A = (0.025\tfrac{m^3}{s})/(\tfrac{\pi}{4}(0.10m)^2) = 3.18\tfrac{m}{s}$$

and

$$V_2 = Q/A_2 = (0.025\tfrac{m^3}{s})/(\tfrac{\pi}{4}(0.05m)^2) = 12.7\tfrac{m}{s}$$

In addition, from the Bernoulli equation,

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2 \text{, or}$$

$$p_2 = p_1 + \tfrac{1}{2}\rho(V_1^2 - V_2^2) = 150\,kP + \tfrac{1}{2}(999\tfrac{kg}{m^3})[(3.18\tfrac{m}{s})^2 - (12.7\tfrac{m}{s})^2]$$

$$= 150\times10^3\tfrac{N}{m^2} - 75.5\times10^3\tfrac{kg\cdot m}{s^2}/m^2 = 74.5\,kPa$$

Thus, from Eq.(1),

$$R_x = 74.5\times10^3\tfrac{N}{m^2}(\tfrac{\pi}{4}(0.05m)^2)\cos 20° - 150\times10^3\tfrac{N}{m^2}(\tfrac{\pi}{4}(0.1\,m)^2)$$
$$+[(12.7\tfrac{m}{s})\cos 20° - 3.18\tfrac{m}{s}](999\tfrac{kg}{m^3})(0.025\tfrac{m^3}{s}) = \underline{\underline{-822\,N}}$$

Similarly, in the y-direction $\int_{cs} v\rho\vec{V}\cdot\hat{n}\,dA = \Sigma F_y$, or

$$(-V_2\sin 20°)\rho V_2A_2 = p_2A_2\sin 20° + R_y$$

or

(2) $$R_y = -V_2\sin 20°\,\dot{m} - p_2A_2\sin 20°$$

$$= -(12.7\tfrac{m}{s})\sin 20°(999\tfrac{kg}{m^3})(0.025\tfrac{m^3}{s}) - 74.5\times10^3\tfrac{N}{m^2}(\tfrac{\pi}{4}(0.05m)^2)\sin 20°$$

$$= \underline{\underline{-156\,N}}$$

5.33

5.33 Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.33. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the x and y components of the force that the pipe exerts on the tee.

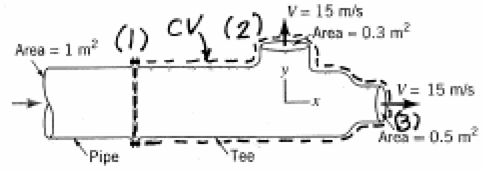


FIGURE P5.33

Use the control volume shown.

For the x-component of the force exerted by the pipe on the tee we use the x-component of the linear momentum equation.

$$-V_1\rho V_1 A_1 + V_3\rho V_3 A_3 = P_1A_1 - P_3A_3 - P_{atm}(A_1 - A_3) + F_x$$

$$= (P_{1_{gage}} + P_{atm})A_1 - (P_{3_{gage}} + P_{atm})A_3 - P_{atm}(A_1 - A_3) + F_x$$

$$= P_{1_{gage}} A_1 + F_x \qquad (1)$$

To get V_1 we use conservation of mass

$$Q_1 = Q_2 + Q_3$$

or $A_1V_1 = A_2V_2 + A_3V_3$

so $V_1 = \dfrac{A_2V_2 + A_3V_3}{A_1} = \dfrac{(0.3\,m^2)(15\,\frac{m}{s}) + (0.5\,m^2)(15\,\frac{m}{s})}{1\,m^2} = 12\,\frac{m}{s}$

To estimate $P_{1_{gage}}$ we use Bernoulli's equation for flow between (1) and (2)

$$\frac{P_{1gage}}{\rho} + \frac{V_1^2}{2} = \frac{P_{2gage}}{\rho} + \frac{V_2^2}{2}$$

$$P_{1gage} = \rho\left(\frac{V_2^2 - V_1^2}{2}\right) = \left(999\,\frac{kg}{m^3}\right)\frac{\left[\left(15\,\frac{m}{s}\right)^2 - \left(12\,\frac{m}{s}\right)^2\right]}{2}\left(1\,\frac{N\cdot s^2}{kg\cdot m}\right)$$

$$P_{1gage} = 40{,}500\,\frac{N}{m^2}$$

Now using Eq.(1) we get:

$$\left[-\left(12\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(12\,\frac{m}{s}\right)(1\,m^2) + \left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.5\,m^2)\right]\left(1\,\frac{N\cdot s^2}{kg\cdot m}\right) =$$

$$\left(40{,}500\,\frac{N}{m^2}\right)(1\,m^2) + F_x$$

or $-72{,}000\,N = F_x$

so $F_x = \underline{\underline{72{,}000\,N}} \leftarrow$

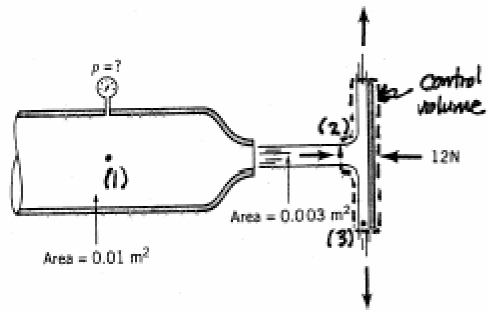
For the y component of the force exerted by the pipe on the tee we use the y component of the linear momentum equation to get

$$V_2\rho V_2 A_2 = F_y$$

$$\left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.3\,m^2) = \underline{\underline{67{,}400\,N}} \uparrow = F_y$$

5.52

5.52 Air flows into the atmosphere from a nozzle and strikes a vertical plate as shown in Fig. P5.52. A horizontal force of 12 N is required to hold the plate in place. Determine the reading on the pressure gage. Assume the flow to be incompressible and frictionless.



■ FIGURE P5.52

To determine the static gage pressure at station (1) we first consider the frictionless and incompressible flow of air from (1) to (2). The Bernoulli equation for this flow is

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} = \overset{0 \text{ gage}}{\cancel{\frac{p_2}{\rho}}} + \frac{V_2^2}{2} \qquad (1)$$

We note that V_1 and V_2 are linked by the continuity (conservation of mass) equation

$$Q_1 = Q_2 \quad \text{or} \quad A_1 V_1 = A_2 V_2 \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{p_1}{\rho} + \frac{\left(\frac{A_2}{A_1} V_2\right)^2}{2} = \frac{V_2^2}{2} \qquad (3)$$

To determine V_2 we use the linear momentum equation for the flow from (2) to (3). For the control volume sketched above the linear momentum principle yields

$$-V_2 \rho V_2 A_2 = -12\,\text{N}$$

or

$$V_2 = \sqrt{\frac{(12\,\text{N})}{\rho A_2}} = \sqrt{\frac{12\,\text{N}}{\left(1.23\,\frac{\text{kg}}{\text{m}^3}\right)\left(1\,\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(0.003\,\text{m}^2\right)}}$$

and

$$V_2 = 57\,\frac{\text{m}}{\text{s}}$$

(con't)

5.52 (con't)

Now, with Eq. 3

$$p_1 = \rho \left[\frac{V_2^2}{2} - \frac{\left(\frac{A_2}{A_1} V_2 \right)^2}{2} \right]$$

or

$$p_1 = \left(1.23 \frac{kg}{m^3}\right)\left(1 \frac{N \cdot s^2}{kg \cdot m}\right) \left\{ \frac{\left(57 \frac{m}{s}\right)^2}{2} - \frac{\left[\left(\frac{0.003\, m^2}{0.01\, m^2}\right)\left(57 \frac{m}{s}\right)\right]^2}{2} \right\}$$

and

$$p_1 = 1820 \frac{N}{m^2} = 1820\ Pa = \underline{\underline{1.82}}\ kPa$$

5.60

5.60 A vertical jet of water leaves a nozzle at a speed of 10 m/s and a diameter of 20 mm. It suspends a plate having a mass of 1.5 kg as indicated in Fig. P5.60. What is the vertical distance h?

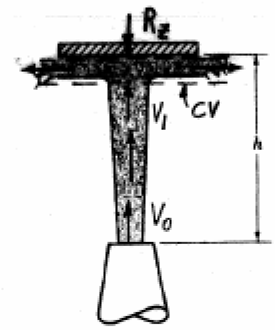


FIGURE P5.60

To determine the vertical distance h we apply the vertical direction component of the linear momentum equation (Eq. 5.22) to the water in the control volume shown in the sketch above. Thus,

$$-R_z - \rho g \forall_{water} = -V_1 \rho A_1 V_1 = -\rho V_1^2 \frac{\pi D_1^2}{4} \qquad (1)$$

The vertical reaction force of the plate on the water is equal in magnitude to the weight of the plate, or

$$R_z = g\, m_{plate} = (9.81\ \tfrac{m}{s^2})(1.5\ kg) = 14.7\ N$$

Also, the weight of the water within the control volume, $\rho g \forall_{water}$, is negligible, and the mass flowrate is

$$\dot{m} = \rho A_1 V_1 = \rho A_0 V_0 = (999\ \tfrac{kg}{m^3})\tfrac{\pi}{4}(0.02 m)^2 (10\ \tfrac{m}{s}) = 3.13\ \tfrac{kg}{s}$$

Thus, Eq 1 becomes

$$-14.7\ N = -V_1 \dot{m} \quad \text{or} \quad V_1 = \frac{14.7\ N}{3.13\ kg/s} = 4.70\ \tfrac{m}{s}$$

From the Bernouli Equation (Eq. 3.7) we have

$$p_0 + \tfrac{1}{2}\rho V_0^2 + \gamma z_0 = p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1, \quad \text{where } p_0 = p_1 = 0$$
$$z_0 = 0,\ z_1 = h$$

Thus,

$$\tfrac{1}{2}\rho V_0^2 = \tfrac{1}{2}\rho V_1^2\ h$$

or since $\gamma = \rho g$

$$h = \frac{1}{2g}(V_0^2 - V_1^2) = \frac{1}{2(9.81\frac{m}{s^2})}(10^2 - 4.70^2)\frac{m^2}{s^2} = \underline{\underline{3.97\ m}}$$